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ELONGATION FLOW OF A VISCOELASTIC JET UNDER CONDITIONS OF EXTERNAL FRICTION

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The problem of a stationary longitudinal flow of a free viscoelastic jet under conditions of friction of its surface is formulated and solved.

Among all operations of the technological process of producing films, orientational elongation is one of the most critical, to a large extent determining the quality and properties of the films.

As a technological example of the problem of a longitudinal flow under conditions of external friction we consider the process of orientation on a machine of traditional type that consists of low- and high-speed groups of thermostated rolls [1]. It is found experimentally that orientational elongation takes place not only in the gap between the rolls but also on the surface of the rolls, thus generating slipping not equal in value on different parts around the circle of the roll [2]. Friction effects in roll elongation are very important because they may cause instability of "stick-slip" type, which to a large extent prevents transition to thin films, for which operation in the mode of irregular slipping is destructive.

In spite of this fact, at present there is no detailed analysis of the physical laws governing the flow, except for uniaxial elongation of a free jet [3]. The first step is to obtain stationary solutions. In the present paper, an analytical solution of the stationary problem is obtained and analyzed with a number of simplifying assumptions.

A schematic diagram of a machine for orientational elongation is given in Fig. 1 [4]. A film 1 that enters the machine at a linear velocity v_{10} is held by pressing rolls 2 from possible slipping and then passes over the surface of rotating rolls 3, where it is heated in the first zone of preliminary heating. Then the film reaches orienting rolls 4, where it is additionally heated by an IR heater 5. The film is elongated in the gap between the two rolls 4 due to the fact that the circumferential velocity v_{3e} of the take-off rolls is higher than v_{10}. On rolls 6 the film is uniformly cooled and it is transported from the machine by pressing rolls 2.

1. Problem Formulation. By the character of motion the region of film residence in the machine is conventionally divided into three zones. The distribution of the velocity v, tension T, and temperature T_- of the film over the zones is shown in Fig. 1. Lines 7 and 8 indicate the circumferential velocities of the heating and cooling rolls, respectively. The tension along the length of the zone of orientation II is constant. The film temperature T_- increases uniformly up to the zone of orientation and then decreases on the cooling rolls. In the subsequent analysis the value of the temperature within each zone is taken to be constant (dashed lines in Fig. 1).

The following assumptions are adopted. The physical properties of the polymer and the parameters of the rheological model correspond to the mean temperature and remain constant within each zone. The film width does not change. The tensile stresses are uniform across the cross section. The forces of inertia, aerodynamic friction, intrinsic weight, and centrifugal forces are much smaller than the forces of rheological resistance. Orienting rolls 4 rotate without friction. Elongation of the film in the gaps between the pressing and heating, (cooling) rolls is neglected. Film friction along the surface of the rolls is described by Amonton's law. Due to the low thermal conductivity of polymers we take the temperature of the contacting surface of the film equal to the temperature of the roll, in accordance with which we adopt the coefficient of friction. Thermal effects (dissipative self-heating, crystallization, heating due to friction) are disregarded. At the junctions of the zones conditions of constant thickness, axial velocity, and tensile stresses are satisfied. The radius of the rolls greatly exceeds the thickness of the film, $R >> \delta$, and therefore its curvature is neglected.

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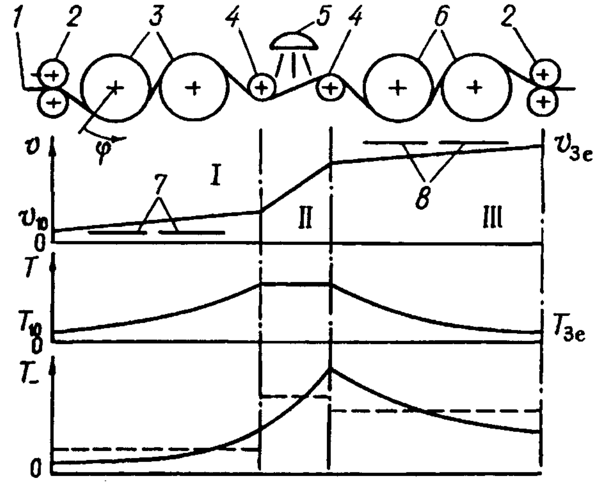

Fig. 1. Schematic diagram of plane film preparation and distribution of the axial velocity v, tension T, and temperature T_-: 1) isotropic film, 2) pressing rolls, 3) slowly rotating heating rolls, 4) orienting rolls, 5) heater, 6) high-speed rotating cooling rolls, 7) circumferential velocity of the heating rolls, 8) circumferential velocity of the cooling rolls.

For the process of orientational deformation to be rather effective, the time of material relaxation should be close to the time of elongation. Therefore, the viscoelastic properties of the polymer are taken into account.

We consider the general case of an isothermal elongation flow of a plane jet. For any of the three zones the rheological equation of the material is

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij}, \tag{1}$$

The deviator stresses obey the equation (the Maxwell model) [5]

$$\tau_{ij} + \lambda\frac{D\tau_{ij}}{Dt} = 2\eta d_{ij}. \tag{2}$$

The case of the upper convective derivative

$$\frac{D\tau_{ij}}{Dt} = \frac{\partial\tau_{ij}}{\partial t} + v_e\frac{\partial\tau_{ij}}{\partial x_e} - \tau_{ej}\frac{\partial v_i}{\partial x_e} - \tau_{ie}\frac{\partial v_j}{\partial x_e}. \tag{3}$$

is considered.

The flow is steady-state (description of motion according to Euler). In all three zones so-called quasi-one-dimensional flow takes place [3]. Friction forces affecting the film from the side of the roll do not cause considerable shear deformations: $|\partial v_i/\partial x_j| \ll |\partial v_i/\partial x_i|$, $|\tau_{ij}| \ll |\tau_{ii}|$, $i \neq j$. The case of considerable shear deformations in a plane jet elongated on the surface of a roll is considered in [6]. The diagonal components of the tensor of deformation velocities are $d_{11} = dv/dx$, $d_{22} = 0$, $d_{33} = -dv/dx$. Here x is the longitudinal coordinate, coinciding with the trajectory of the film; v is the axial velocity.

With account for the condition $\sigma_{33} = 0$ we have from (1) $p = \tau_{33}$, $\sigma_{11} = \tau_{11} - \tau_{33}$, $\sigma_{22} = \tau_{22} - \tau_{33}$. For the mode of a steady-state flow $\partial/\partial t = 0$, Eq. (2) with account for (3) decomposes to the three equalities

$$\tau_{11} + \lambda\left(v_1\frac{\partial\tau_{11}}{\partial x_1} + v_2\frac{\partial\tau_{11}}{\partial x_2} + v_3\frac{\partial\tau_{11}}{\partial x_3} - 2\tau_{11}\frac{\partial v_1}{\partial x_1}\right) = 2\eta\frac{\partial v_1}{\partial x_1},$$

$$\tau_{22} + \lambda\left(v_1\frac{\partial\tau_{22}}{\partial x_1} + v_2\frac{\partial\tau_{22}}{\partial x_2} + v_3\frac{\partial\tau_{22}}{\partial x_3} - 2\tau_{22}\frac{\partial v_2}{\partial x_2}\right) = 2\eta\frac{\partial v_2}{\partial x_2},$$

$$\tau_{33} + \lambda \left(v_1 \frac{\partial \tau_{33}}{\partial x_1} + v_2 \frac{\partial \tau_{33}}{\partial x_2} + v_3 \frac{\partial \tau_{33}}{\partial x_3} - 2\tau_{33} \frac{\partial v_3}{\partial x_3} \right) = 2\eta \frac{\partial v_3}{\partial x_3} .$$

Substracting the third equation from the first equation, we obtain for the longitudinal stress

$$\sigma_{11} + \lambda \left(v_1 \frac{\partial \sigma_{11}}{\partial x_1} + v_2 \frac{\partial \sigma_{11}}{\partial x_2} + v_3 \frac{\partial \sigma_{11}}{\partial x_3} - 2\tau_{11} \frac{\partial v_1}{\partial x_1} + 2\tau_{33} \frac{\partial v_3}{\partial x_3} \right) = 2\eta \left(\frac{\partial v_1}{\partial x_1} - \frac{\partial v_3}{\partial x_3} \right) .$$

With account for the uniformity of the axial stresses over the thickness and width of the film $\sigma_{11}/\partial x_2 = \sigma_{11}/\partial x_3 = 0$, the continuity condition $\partial v_1/\partial x_1 = -\partial v_3/\partial x_3$, and the relation $\tau_{11} = \sigma_{11} + \tau_{33}$ and assuming $|\sigma_{11}| >> 2|\tau_{33}|$, in final form we have for the axial tensile stress σ_{11} in any of the three zones

$$\sigma_{11} + \lambda \left(v_1 \frac{\partial \sigma_{11}}{\partial x_1} - 2\sigma_{11} \frac{\partial v_1}{\partial x_1} \right) = 4\eta \frac{\partial v_1}{\partial x_1} . \tag{4}$$

The distribution of film tension along the arc of its contact with the rolls in zones I and III can be found by the Euler approach [7]. For the first zone, the equilibrium of a film element of length $Rd\varphi$ on the surface of the heating roll is described by the equations

$$\tau_{N1}\, Rb\, d\varphi = dT_1 , \ \ \Sigma_1 Rb = T_1 , \ \ \tau_{N1} = f_1 \Sigma_1 ,$$

$$\varphi = 0 : \ T_1 = T_{10} ; \ \ \varphi = \varphi_e : \ T_1 = T_{1e} .$$

Similarly, for zone III

$$\tau_{N3}\, Rb\, d\varphi = - dT_3 , \ \ \Sigma_3 Rb = T_3 , \ \ \tau_{N3} = f_3 \Sigma_3 ;$$

$$\varphi = 0 : \ T_3 = T_{1e} ; \ \ \varphi = \varphi_e : \ T_3 = T_{3e} . \tag{5}$$

The solution of these problems has the form

$$T_1 = T_{10} \exp (f_1 \varphi) , \ \ T_3 = T_{1e} \exp (- f_3 \varphi) . \tag{6}$$

According to (6), heating 3 and cooling 6 rolls "unload" pressing rolls 2, thus reducing the pull force on them.

With account for Eqs. (4), (6), the distributions of the tension T_1, axial velocity v_1, film thickness δ_1, and tensile stresses σ_1 in the first zone are described by the system of equations

$$\begin{gathered} T_1 = T_{10} \exp (f_1 \varphi) , \ \ T_1 = b\delta_1 \sigma_1 , \ \ b\delta_1 v_1 = Q , \\ \sigma_1 + \lambda_1 \left(\frac{v_1}{R} \frac{d\sigma_1}{d\varphi} - \frac{2\sigma_1}{R} \frac{dv_1}{d\varphi} \right) = \frac{4\eta_1}{R} \frac{dv_1}{d\varphi} , \\ \varphi = 0 : \ T_1 = T_{10} , \ \ v_1 = v_{10} , \ \ \delta_1 = \delta_{10} , \ \ \sigma_1 = \sigma_{10} , \\ \varphi = \varphi_e : \ T_1 = T_{1e} , \ \ v_1 = v_{1e} , \ \ \delta_1 = \delta_{1e} , \ \ \sigma_1 = \sigma_{1e} . \end{gathered} \tag{7}$$

Here the second expression determines the pull force; the third – the condition of constancy of the flow rate. $R\varphi$ is used as the longitudinal coordinate.

In the second zone the pull force is constant along the length of the orientation section, and the problem is described by the equations

$$T_{1e} = T_2 = b\delta_2 \sigma_2 = \text{const} , \ \ b\delta_2 v_2 = Q ,$$

$$\sigma_2 + \lambda_2 \left(v_2 \frac{d\sigma_2}{dx} - 2\sigma_2 \frac{dv_2}{dx} \right) = 4\eta_2 \frac{dv_1}{dx}, \tag{8}$$

$$x = 0: \; T_2 = T_{1e}, \; v_2 = v_{1e}, \; \delta_2 = \delta_{1e}, \; \sigma_2 = \sigma_{1e},$$

$$x = l: \; T_2 = T_{1e}, \; v_2 = v_{2e}, \; \delta_2 = \delta_{2e}, \; \sigma_2 = \sigma_{2e}.$$

The problem of flow in the third zone is similar to the first (7) and is presented as

$$T_3 = T_{1e} \exp(-f_3\varphi), \; T_3 = b\delta_3\sigma_3, \; b\delta_3 v_3 = Q,$$

$$\sigma_3 + \lambda_3 \left(\frac{v_3}{R} \frac{d\sigma_3}{d\varphi} - \frac{2\sigma_2}{R} \frac{dv_3}{d\varphi} \right) = \frac{4\eta_3}{R} \frac{dv_3}{d\varphi},$$

$$\varphi = 0: \; T_3 = T_{1e}, \; v_3 = v_{2e}, \; \delta_3 = \delta_{2e}, \; \sigma_3 = \sigma_{2e},$$

$$\varphi = \varphi_e: \; T_3 = T_{3e}, \; v_3 = v_{10}K, \; \delta_3 = \delta_{10}/K, \; \sigma_3 = \sigma_{3e}. \tag{9}$$

2. Solution of the Problems. We introduce dimensionless variables and parameters (the scales are taken at the initial cross section):

$$V_{\{1,2,3\}} = \frac{v_{\{1,2,3\}}}{v_{10}}, \; K = \frac{v_{3e}}{v_{10}}, \; T_{+\{1,2,3\}} = \frac{T_{\{1,2,3\}}}{T_{10}}, \; X = \frac{x}{l}, \; A = \frac{4v_{10}\eta_1}{R\sigma_{10}},$$

$$N_2 = \frac{\eta_2 R}{\eta_1 l}, \; N_3 = \frac{\eta_3}{\eta_1}, \; \mathrm{We}_{\{1,3\}} = \frac{\lambda_{\{1,3\}} v_{10}}{R}, \; \mathrm{We}_2 = \frac{\lambda_2 v_{10}}{l}. \tag{10}$$

With account for (10) the solution of problems (7)-(9) has the form:
1) flow in the first zone ($0 \le \varphi \le \varphi_e$, $1 < V_1 \le V_{1e}$)

$$\exp(f_1\varphi) = Z_1 + Af_1 \left[1 - Z_1 (1 + \mathrm{We}_1 f_1 V_1) \ln (Z_1/V_1)\right],$$

$$Z_1 = (1 + \mathrm{We}_1 f_1 V_1)/(1 + \mathrm{We}_1 f_1), \; T_{+1} = \exp(f_1\varphi);$$

2) flow in the zone of orientation ($0 \le X \le 1$, $V_{1e} \le V_2 \le V_{2e}$)

$$X = (AN_2/T_{+1e}) \ln (V_2/V_{1e}) + \mathrm{We}_2 (V_2 - V_{1e}),$$

$$T_{+2} = T_{+1e} = \exp(f_1\varphi_e) = \mathrm{const};$$

3) flow on the cooling rolls ($0 \le \varphi \le \varphi_e$, $V_{3e} \le V_3 \le K$)

$$\exp(-f_3\varphi) = Z_3 - (AN_3 f_3/T_{+1e}) \left[1 - Z_3 - (1 - \mathrm{We}_3 f_3 V_3) \ln |Z_3 V_{2e}/V_3|\right],$$

$$T_{+3} = T_{+1e} \exp(-f_3\varphi), \; Z_3 = (1 - \mathrm{We}_3 f_3 V_3)/(1 - \mathrm{We}_3 f_3 V_{2e}), \tag{11}$$

where $V_{1e} = V_1 \; (\varphi = \varphi_e)$, $V_{2e} = V_2 \; (X = 1)$, $K = V_{3e} = V_3 \; (\varphi = \varphi_e)$.

The distributions of the velocities $V_1(\varphi)$, $V_2(X)$, $V_3(\varphi)$ and the velocity at the junctions of the zones V_{1e}, V_{2e}, V_{3e} were found by the Newton iteration method. The constant A characterizes the initial tensile stress σ_{10} and is unambiguously related to the take-off velocity (the elongation ratio).

3. Numerical Analysis. An analysis of the mathematical model of a stationary flow is made for the case of orientational elongation of a polyethylene film (low-density polyethylene). The following values of the parameters were used: $\lambda_1 = 10$ sec, $\eta_1 = 10^6$ Pa·sec, $f_1 = 0.6$, $R = 0.2$ m, $\varphi_e = 6$ rad (R and φ_e for the cooling and heating rolls

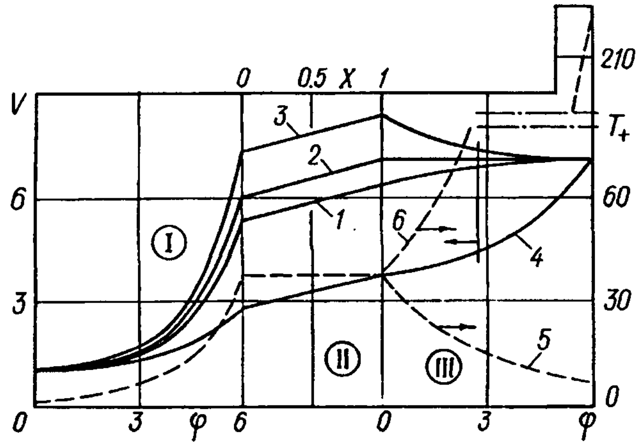

Fig. 2. Distribution of the dimensionless velocity V and pull force T_+ on the heating rolls (section I), in the zone of orientation (II), and on the cooling rolls (III) for $\mathrm{We}_1 = 5$, $\mathrm{We}_2 = 1$: 1) $\mathrm{We}_3 = 0.1$, 2) 0.4761, 3) 1, 4) 1, 5) $S = 1$, 6) $S = -1$.

are identical), $\lambda_2 = 1$ sec, $\eta_2 = 10^5$ Pa·sec, $l = 0.1$ m, $v_{10} = 0.1$ m/sec, $\lambda_3 \sim 1$ sec, $\eta_3 = 10^7$ Pa·sec, $f_3 = 0.3$, $K = 7$, $\mathrm{We}_1 = 5$, $\mathrm{We}_2 = 1$, $A = 60$–180, $N_2 = 0.2$, $N_3 = 10$.

Figure 2 shows the distribution of the dimensionless velocity V and pull force T_+ along the length of the film in all three zones of flow. The effect of the dimensionless relaxation time We_3 (the Weissenberg number) on the flow in zone III is illustrated: $\mathrm{We}_3 = 0.1$ ($A = 86.148$) corresponds to curve 1, $\mathrm{We}_3 = 0.4761$ ($A = 76.318$) to curve 2, $\mathrm{We}_3 = 1$ ($A = 60.874$) to curve 3, $\mathrm{We}_3 = 1$ ($A = 179.814$) to curve 4. The values of the parameter A (in parentheses) provide an assigned elongation ratio $K = 7$. The dashed lines show the distribution of T_+: curve 5 corresponds to first three cases (see the data for curves 1-3) and curve 6 to the fourth case (see the data for curve 4). Curves 4 and 6 will be discussed in what follows.

An analysis of the flow parameters in the first two zones showed that the character of the distribution of V_1, V_2, T_{+1} does not change. The degree of elongation on the heating rolls decreases (V_{1e} becomes smaller) with decrease in We_1 and increase in f_1. A decrease in We_2 facilitates growth of the degree of film elongation in the zone of orientation.

A more complex flow pattern occurs in zone III. To explain the specific features of the flow on the cooling rolls, on the basis of (9) and (10) we write a differential equation for the axial velocity:

$$\frac{dV_3}{d\varphi} = \frac{T_{+1e} V_3 S_+}{\mathrm{We}_3 V_3 T_{+1e} + A N_3 \exp(f_3 \varphi)}, \quad S_+ = (1 - \mathrm{We}_3 f_3 V_3). \tag{12}$$

Equation (12) has the following special feature of the solution: if $\varphi = 0$, $dV_3/d\varphi = 0$, then $V_3 = K = \mathrm{const}$. In the general case the axial velocity in zone III can grow monotonically ($dV_3/d\varphi > 0$), decrease ($dV_3/d\varphi < 0$), or maintain a constant value. Here sgn ($dV_3/d\varphi$) = sgn (S_+), since the function $V_3(\varphi)$ has no extremuma inside the flow zone.

The character of the distribution of V_3 depends on the sign of the function S_+; we can distinguish four flow modes.

1. The time of film residence on the rolls exceeds the time of relaxation, and the condition $S_+ > 0$ is satisfied. The velocity of the film on cooling rolls 6 increases monotonically, reaching the velocity of take-off rolls 2 at the end of the flow zone (see Fig. 1). Effective sliding of the film along the surface of the rolls takes place (curve 1 in Fig. 2). For a Newtonian fluid $\mathrm{We}_3 = 0$, $V_{2e} < K$ always holds, and the distribution of the axial velocity is described by the expression

$$V_3 = V_{2e} (K/V_{2e})^{[1-\exp(-f_3\varphi)]/[1-\exp(-f_3\varphi_e)]}.$$

2. Having increased the time of relaxation, for example, by reducing the temperature of the cooling rolls, we can obtain fulfillment of the condition $S_+ = 0$. Deformations in the film stop: its axial velocity is equal to the take-off velocity (curve 2 in Fig. 2). If the circumferential velocities of the cooling rolls are equal to the take-off velocity, then the film seemingly "sticks" to the surface of the rolls.

3. With a further increase in the time of relaxation of the material in zone III (or with a reduction in the residence time) we can achieve fulfillment of the condition $S_+ < 0$. The axial velocity decreases (curve 3 in Fig. 2), which is associated with elastic recovery (shrinkage) of the film after its elongation. Elastic energy, which is accumulated in the material from the instant of the start of deformation to the instant of contact with the first cooling roll, is released. However, to realize this flow the cooling rolls, equipped with individual drives, should have a circumferential velocity exceeding the axial velocity of the film at the end of the second zone v_{2e} or equal to it (here $v_{2e} > v_{10} \sim K$). From the technological point of view this mode is undesirable since the orientation of the chains in the polymer decreases.

It is important to note that in all three modes relaxation of stresses takes place, and the distribution of the pull force (curve 5 in Fig. 2) is described by the function $T_{+3} = T_{+1e} \exp(-f_3\varphi)$.

The case where the circumferential velocity of the cooling rolls is equal to the take-off velocity is considered. Consequently, the distribution of the axial velocity in the third mode (curve 3 in Fig. 2) contradicts the physical essence of the problem: the film velocity cannot exceed the take-off velocity. The inadequacy of the mathematical model of zone III is caused by the inaccuracy of the notation of Amonton's law in the formulation of the third problem (5): the form used does not allow for the direction of the velocity of slipping. Therefore, it is more correct to write Amonton's law for the third problem in the form

$$\tau_{N3} = f_3 \Sigma_3 S, \quad S = \operatorname{sgn}[v_{3e} - v_3(\varphi)]. \tag{13}$$

In the first zone the condition $S = 1$ is always satisfied, and therefore problem (7) does not require refinement.

With account for (13) the solution of problems (5), (9) has the form

$$0 \le \varphi \le \varphi_e, \quad V_{2e} \le V_3 \le K,$$
$$\exp(-f_3 S\varphi) = Z_3 - (SAN_3 f_3/T_{+1e})\,[1 - Z_3 - (1 - \mathrm{We}_3 f_3 V_3)\ln|Z_3 V_{2e}/V_3|\,], \tag{14}$$
$$Z_3 = (1 - S\mathrm{We}_3 f_3 V_3)/(1 - S\mathrm{We}_3 f_3 V_{2e}), \quad T_{+3} = T_{+1e}\exp(-Sf_3\varphi).$$

By virtue of the mentioned property of Eq. (12), the initial value $S = \operatorname{sgn}[S_+(V_{2e})]$, which is retained along the entire length of zone III, can be used as S.

4. Solution (14) for $S = -1$ describes the fourth mode of the flow in zone III, and for $S = 1$ the third mode. Results of an analysis of Eqs. (14) are presented in Fig. 2 (curves 4 and 6). The character of the distribution of the axial velocity and the pull force changes considerably in this mode: the region of intense deformations shifts from the zone of orientation II to the pressing rolls, and the film tension increases exponentially, $S = -1$, $T_{+3} = T_{+1e}\exp(f_3\varphi)$ (curve 6 in Fig. 2), reaching the highest value near the pressing rolls, which literally "peel off" the film from the cooling rolls. If we take into account the friction of rest, then the tension jump will be more substantial. With identity of the Weissenberg number for curves 3 and 4 the initial stress σ_{10} decreases in transition from the third to the fourth mode because the parameter A increases.

In transition from the second to the fourth mode the pull force at the pressing rolls experiences a jumpwise change from $T_{+1e}\exp(-f_3\varphi_e)$ to $T_{+1e}\exp(f_3\varphi_e)$, which under real conditions should cause rupture of the film at the pressing rolls or onset of self-oscillations in the system. Even if rupture does not take place, the film obtained in the fourth mode will have low physicomechanical characteristics since elongation occurs at a lowered temperature.

Thus, the second mode ($S = 0$) can be considered as a boundary mode. It separates two qualitatively different stationary flows of a plane viscoelastic jet: subcritical (the first mode $S = 1$) and postcritical (the fourth mode $S = -1$). The first mode, in which the elastic properties of the material of the film manifest themselves slightly, is technologically acceptable. The second mode (critical) is rather unstable to small perturbations, for example, of the thickness, velocity, tension, etc. In this case a jumpwise transition to the fourth mode with subsequent rupture of the film is probable.

In dimensional form the condition of flow in the first mode can be written as $\lambda_3 K v_{10} f_3/R < 1$. Hence it follows that in order to provide stable elongation of the film (or transition to a stable mode) the temperature of the cooling rolls can be increased (reduction of λ_3), the efficiency can be decreased (reduction of v_{10} and K), the coefficient of friction can be reduced, and the radius of the cooling rolls can be increased.

The phenomena associated with instability of the process of orientational elongation are caused by nonlinear effects of the flow on the cooling rolls. The prerequisites for onset of self-oscillations in the system are: a small amplitude of disturbances sufficient for jumpwise transition from the second mode to the fourth and back; a considerable change in the pull force accompanying this transition; finite "rigidity" of the drive of pressing rolls 2 (see Fig. 1). By "rigidity" we mean the ability of the pressing rolls to preserve the take-off velocity with a change in the film tension.

The mechanism of onset of self-oscillations is the following. Let a flow corresponding to the second mode be organized. We assume that the take-off velocity v_{3e} increased slightly. The flow passes over to the fourth mode. The film tension at the pressing take-off rolls increases sharply, thus causing, due to the finite "rigidity" of the drive, a decrease in the take-off velocity. The take-off velocity decreases to the level corresponding to the second mode of the flow. Here a reciprocal transition from the fourth to the second mode takes place, which is accompanied by a jumpwise decrease in the pull force T_{3e}. The decrease in the load on the pressing rolls due to the finite "rigidity" of the drive increases the take-off velocity, and thus conditions for a "jump" of the flow from the second to the fourth mode etc. are produced repeatedly. The frequency of the self-oscillations is determined by the ratio of the longitudinal "elasticity" of the film and the moment of inertia of the pressing rolls with the drive. Probably, precisely this is the mechanism of the "stick-slip" instability noted in [2].

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NOTATION

σ_{ij}, components of the stress tensor; p, isotropic pressure; δ_{ij}, Kronecker symbol; τ_{ij}, deviator components of the stress; T_{-}, temperature; d_{ij}, deviator part of the tensor of deformation rates; η, coefficient of shear viscosity; λ, time of relaxation; η_1, η_2, η_3, coefficients of shear viscosity for zones I, II, III; λ_1, λ_2, λ_3, time of relaxation for zones I, II, III; D/Dt, convective derivative according to Oldroyd; T_1, T_2, T_3, film tension in zones I, II, III; T_{1e}, T_{2e}, T_{3e}, same, at the end of zones I, II, III; τ_{N1}, τ_{N3}, shear stresses caused by film friction against the surface of the heating and cooling rolls, respectively; Σ_1, Σ_3, normal pressures from the side of the film on the roll; T_{10}, initial tension of the film at the pressing rolls (see Fig. 1); b, thickness of the film; $v_1(\varphi)$, $v_2(x)$, $v_3(\varphi)$, axial velocity in zones I, II, III; v_{1e}, v_{2e}, v_{3e}, same, at the end of zones I, II, III; v_{10}, axial velocity at the inlet to the first zone; $\delta_1(\varphi)$, $\delta_2(x)$, $\delta_3(\varphi)$, thickness of the film in zones I, II, III; δ_{1e}, δ_{2e}, δ_{3e}, same, at the end of zones I, II, III; σ_{10}, initial thickness of the film; $\sigma_1(\varphi)$, $\sigma_2(x)$, $\sigma_3(\varphi)$, tensile stresses in zones I, II, III; σ_{1e}, σ_{2e}, σ_{3e}, same, at the end of zones I, II, III; σ_{10}, tensile stress at the inlet to the first zone; φ_e, total angle of contact of all rolls (heating or cooling); φ, angle reckoned from the line of contact of the film with the roll surface; R, radius of the rolls; f_1, f_3, coefficients of sliding friction of the film on the heating and cooling rolls; Q, bulk flow rate of the polymer; x, longitudinal coordinate reckoned from the line of descent of the film from the last heating roll; l, length of the zone of orientational elongation; $V_{\{1,2,3\}}$, dimensionless velocity in zones I, II, III; K, "machine" ratio of elongation; $T_{+\{1,2,3\}}$, dimensionless pull force in zones I, II, III; $T_{+\{1,2,3\}e}$, dimensionless pull force at the end of zones I, II, III; X, dimensionless longitudinal coordinate in the zone of orientation; A, N_2, N_3, dimensionless parameters; $\mathrm{We}_{\{1,2,3\}}$, Weissenberg number for zones I, II, III; V_{1e}, dimensionless velocity at the end of zone I; V_{2e}, same, at the end of zone II; V_{3e}, same, at the end of zone III. Subscripts: i, j = 1, 2, 3: 1 corresponds to the unit vector of the tangent to the axis of the film, 2 to the unit vector of the binormal, 3 to the unit vector of the normal; e corresponds to the variable at the end of the zone of the flow. Numerical indices of variables and parameters indicate the number of the zone of the flow (of the problem).

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